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Communication and Connection in solving Differential Equation: Analysis of Factors affecting the understanding of Mathematical concepts and Knowledge of Technical and Engineering students in Solving Differential Equations

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COMMUNICATION AND CONNECTION IN SOLVING DIFFERENTIAL EQUATION: ANALYSIS OF FACTORS AFFECTING UNDERSTANDING OF MATHEMATICAL CONCEPTS AND KNOWLEDGE IN SOLVING DIFFERENTIAL EQUATION

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ABSTRACT

This study emphasizes the significance of connections and communication in solving differential equation problems, highlighting their impact on meaningful learning. The National Council of Teachers of Mathematics (NCTM) emphasizes these process standards, advocating for curricula that interlink various mathematical topics and underscore their practical applications. Similarly, the Islamic Republic of Iran's curriculum incorporates these principles. Despite their recognized importance, educators and students often overlook these concepts, leading to superficial understanding and inadequate preparation for advanced challenges. The participants in this study are 30 engineering students from the Islamic Azad University who attended a course on differential equations during the first semester of the 2022-2023 academic year. Over six consecutive weeks, the students were taught how to solve first-order differential equations, and their learning was assessed. The findings revealed that most students struggled to apply previously learned material to differential equations, indicating a deficiency in connecting new concepts with prior knowledge, such as simplifying algebraic expressions and factoring.

Keywords: Connections and communication, problem-solving, differential equations, conceptual learning in mathematics.

Introduction

Conceptual knowledge is one of the essential elements and necessary skills for students to become familiar with and solve various challenges they will face in the future. Conceptual knowledge in mathematics helps individuals understand and master social, economic, and natural issues (Van De Ville, 2003). The National Curriculum Document of the Islamic Republic of Iran (2016) clearly states that one of the important aspects of mathematics is to empower individuals to describe and control complex material, natural, economic, and social situations. Strengthening the learner's conceptual mathematical knowledge is essential to achieve this goal.

The National Council of Teachers of Mathematics (NCTM) in the United States also emphasizes that students should learn mathematics based on conceptual understanding and actively construct new understandings from their previous experiences and knowledge. This principle is based on two main ideas: first, that learning mathematics with understanding is necessary, and second, that students can learn mathematics based on conceptual understanding. One way to increase conceptual knowledge is to pay attention to understanding the connections between mathematical concepts; understanding is the quantitative and qualitative connection between ideas an individual makes with their ideas. The benefits of this type of understanding include memory growth, creativity and development, problem-solving ability, reduced need to memorize information, helping to learn new concepts and patterns, and improved beliefs (Van De Ville, 2003). To achieve meaningful learning, it is necessary to relate new knowledge to previously learned concepts and use those (Karimi et al., 2017).

In new teaching-learning approaches, meaningful learning is emphasized. According to this theory, first proposed by Ausubel in 1963 (Seif, 2002), meaningful learning occurs when the learner can relate new knowledge to the existing cognitive network in their mind (Yew, 2018). Research shows that in developing the cognitive structure of the mind, students are forced to establish connections between concepts. Through these connections, their understanding of mathematics improves, and they understand mathematics as a set of coordinated and not separate concepts (Bartletz, 1995). By gaining their own experiences, students develop various mathematical concepts, some of which can be expanded as new ideas, links between concepts, and even the connection between one concept and concepts in other subjects; this ability is called mathematical communication skill. Students who connect mathematical ideas and concepts will achieve deep and lasting understanding (NCTM, 2000).

The ability to establish mathematical connections and communication between mathematical topics and mathematics and other disciplines by students is one of the main goals of the mathematics learning process. Because mathematics is related to the real world and everyday life, teachers should allow students to discover these relationships; in this way, students will be more successful in learning mathematics (Rohendi & Dalpaja, 2013). To achieve the deep and broad understanding we expect, establishing connections between mathematical topics and other learning areas is important (Butler, 2005). The National Council of Teachers of Mathematics (NCTM) has introduced connections and communication as one of the main standards in mathematics education. This council has stipulated that educational programs from preschool to twelfth grade should enable students to identify and use the connections between mathematical ideas, understand how these ideas relate to each other to form a coherent whole, and recognize the presence of mathematics in structures and contexts outside the school environment and use it.

According to these standards, educational programs should be designed so that learners can recognize and utilize the connections and communication between mathematical topics. They should understand the role and connection of each mathematical concept in forming a coherent whole and use mathematics outside the school environment. In this way, they can understand mathematics more broadly and practically and use it in daily life and other disciplines (Kilpatrick, 2002).

Butler (2005) believes connections often do not occur by chance, and many students cannot recognize them independently. Therefore, teachers should have appropriate programs to identify these connections. These programs can help improve learning and enhance teaching efficiency. Engineering is one of the branches of science that has a serious connection with mathematics (Khiat, 2010). Therefore, the quality of mathematics education in the engineering education system is expected to receive more attention. Despite the strengths of engineering education in Iran, there are numerous shortcomings (Memarian, 2011). Despite the importance of mathematics education in the quality of engineering education, only a few studies have been conducted on university mathematics education, especially studies related to teaching differential equations to engineering students (Araújo, 2010). In the Iranian university education system, teaching the differential equations course, due to its prerequisite nature for specialized courses and its practical application, is of great importance for the academic success of many engineering students (Karimi & Fardinpour, 2012). Enthusiasts of engineering education and academic mathematics education specialists have also emphasized student-centered education to reduce student's indifference to education (Memarian & Hossein, 2011).

In this study, the researchers intend to track the weaknesses of engineering students in the differential equations course and, by evaluating their performance in solving differential equations, help clarify the points where students have problems connecting and relating to prerequisite concepts. Various studies have examined the role of the standard of the connection in students' self-confidence (Sudia & Muhammad, 2020), self-regulated learning, problem-solving, and mathematical problem-posing (Najoan et al., 2024). However, in the present study, the researchers intend to examine the conceptual knowledge of several engineering students from the perspective of the connections and communication of mathematical concepts in the differential equations course. For this purpose, the students' performances are evaluated from four perspectives: recognizing the method of solving the differential equation, Accuracy in mathematical relationships, Proper use of mathematical relationships, and maintaining coherence in solving the differential equation.

Research Background

Various studies have investigated student's conceptual knowledge by assessing connections between mathematical concepts. Most of these studies are dedicated to school mathematics education. For example, a study titled "The Role of Mathematical Connections in Mathematical Problem Solving" claimed that mathematical connections play an important role in students solving mathematical problems. This research emphasizes that students with appropriate mathematical communication skills solve mathematical problems well, while those with weak ones fail to solve them (Pambudi et al., 2020). However, the study of Kenedi et al. (2019) aimed to determine the mathematical connection ability of elementary school students to solve mathematical problems, and the results showed that the mathematical connection ability of elementary school students to solve mathematical problems is low.

Putri & Wutsq (2017) examined in their study titled "Student's Mathematical Connection Ability in Solving Real-world Problems" that the mathematical connection ability of eighth-grade students in solving real-world problems is at a low level. Most of their issues in solving real-world problems are limited to understanding the problem and connecting it to mathematical concepts. The target group for another study is also junior high school (ninth-grade students). This research examines the effectiveness of Think Talk Write (TTW) learning in improving students' mathematical communication Ability. The result showed that for students to think critically, calculate, reason, and be able to analyze a problem, they must enhance their mathematical communication skills, which are related to everyday life through mathematics learning (Kamaruddin et al., 2023).

In addition, Karakoç and Alacac (2015), in their research "Real World Connections in High School Mathematics Curriculum and Teaching," examined the reasons for using real-world connections in mathematics education based on experts' opinions. They reported on the advantages, disadvantages, and examples of real-world connections. The article titled "Efforts to Improve Mathematical Communication Skills in Mathematics Learning in

Indonesia" explores various methods and strategies for enhancing mathematical communication. It employs the PRISMA method to review 30 publications and identifies Realistic Mathematics Education (RME), cooperative learning models, and media use as the most common approaches. It highlights middle school education and geometry topics as frequently researched areas. This study provides valuable insights for educators aiming to enhance mathematical communication (Epih, 2024).

Examining conceptual mathematical knowledge using connections and communication is not limited to school education and has also attracted the attention of researchers in higher education. For example, Widjajanti (2013), in a case study on mathematics education students at Yogyakarta State University in Indonesia, has examined the ability of fourth-semester students in the discrete mathematics course in writing expressions, reasons, and explanations, as well as using terms, symbols, tables, charts, diagrams, and mathematical models. The results show that students have weaknesses in writing reasons and using charts and mathematical models.

Junarti et al. (2019), also in this field, in their research about the profile of structure sense in abstract algebra Instruction in Indonesian Mathematics Education, examined the inability to recognize the structure of set elements, operation symbols, and the characteristics of binary operations in group proof structure. They concluded that structural sense should be learned to help understand and create connections in abstract algebra.

The closest study to the aim of this research is one conducted by Camacho et al. (2012). The authors of this article consider one of the fundamental challenges for engineering students in the algebraic approach to teaching differential equations to be the selection of the most appropriate solution method for solving equations. After correctly identifying the type of a differential equation, sometimes it is necessary to transform the differential equation from one form to another, both of which are equivalent. This is because some differential equations can be solved more easily and quickly than others. Camacho refers to the error of "transforming a differential equation from one form to another" and the error of "knowing the solution algorithms but lacking sufficient knowledge on how and when to use these algorithms." The combination of these two errors is equivalent to what Vat calls a "second-stage error," known as "recalling the most appropriate solution." In other words, students have weaknesses in connecting and linking the main concepts of differential equations. This means that while they might be able to identify the appropriate algorithm for a differential equation, they have weaknesses in solving it using algebraic methods. This weakness stems from a deficiency in the connections and linkages of concepts among engineering students in differential equations courses (Moradi et al., 2023).

Moradi et al. (2023) examined the errors and misunderstandings of first-year engineering students in a case study. They concluded that most errors were algebraic and conceptual, rooted in an inadequate understanding of mathematical concepts from high school. In other words, if students do not have weaknesses in mathematical connections and communication, they will make fewer algebraic and conceptual errors.

A review of the above studies shows that most research in connection and communication has been conducted in school education. Therefore, in the present study, the researchers intend to focus on higher education and aim to explore the conceptual understanding of several engineering students through the lens of connections and communication of mathematical concepts in a differential equations course. The students' performances will be evaluated based on four key aspects: identifying the method for solving the differential equation, accuracy in mathematical relationships, appropriate application of mathematical principles, and maintaining coherence throughout the solution process. They will examine the reasons for students' weaknesses in solving differential equations.

Methodology

This research was conducted within a quantitative paradigm designed to identify the weaknesses of engineering students in conceptual knowledge of solving differential equations. The participants in this study were a convenient sample of 30 undergraduate engineering students (both male and female) from various engineering disciplines at Islamic Azad University during the second semester of the 2022-2023 academic year. The data collection tool was a test comprising six questions on differential equations, designed in collaboration with mathematics education specialists and mathematicians, and their content validity was confirmed (Appendix 1).

The first three questions used common differential equation methods, such as homogeneous, exact, and first-order, requiring students to apply concepts from high school and university-level calculus. The remaining three questions involved non-homogeneous, inexact, and Bernoulli methods, building upon the fundamental concepts and methods of the earlier questions.

Students underwent six consecutive weeks of instruction on first-order differential equations, and their performance was evaluated using the test. The test was graded by two scorers for consistency, and the results were analyzed and categorized to identify weaknesses in four key areas based on the framework of Junarti (2019):

1. Correctness in using the method to solve differential equations.

2. Accuracy in mathematical relationships (precision in writing mathematical relations and formulas).
3. Proper use of mathematical relationships.
4. Coherence in solving differential equation problems (consistency in solving differential equation problems).

Each question was scored on a scale of 1. Table 1 shows the analytical framework for the test questions.

Table 1: Analytical Framework for Test Questions

Question	S-1	S-2	S-3	S-4
1: Homogeneous differential equation	Recognition of homogeneity	Apply variable change $u = \frac{y}{x}$	Convert to separable differential equation	Solving the integral
2: Complete differential equation	Determining dx and dy coefficients	Partial derivative and detection of completeness	The method of obtaining u(x,y)=c as the solution of the equation	Solving the integral
3: First-order differential equation	First order diagnosis	Find p(x), q(x)	Placement in the solution formula of the first-order equation	Solving the integral
4: Nonhomogeneous differential equation	Heterogeneous diagnosis	Convert nonhomogeneous to homogeneous	Transform into a separable differential equation	Solving the integral
5: Incomplete differential equation	Incomplete diagnosis	Find the right invoice.	Placement in the integral factor formula and solving the following integral expression	Solving the integral
6: Bernoulli differential equation	Identifying the type of equation and obtaining n	Convert Bernoulli's equation to first order equation	Placement in the solution formula of the first-order equation	Solving the integral

In the table, S1, S2, S3, S4 represent the stages of solving differential equations.

- S1: Stage of identifying the type of equation.
- S2: Accuracy and precision of mathematical relationships.
- S3: Maintaining coherence in the process of solving differential equation problems.
- S4: Correct use of mathematical relationships.

The test questions were designed to be interrelated, and the student needs to identify this connection and communication in the questions. For example, question 1, which is about solving a homogeneous equation, is related to question 4, which is about solving a non-homogeneous equation. To solve it, the student must first transform the non-homogeneous equation into a homogeneous one.

Table 2: Mean Scores of Differential Equations Questions

Question		S-1			S-2			S-3			S-4		
		Q-1	Q-2	Q-3	Q-1	Q-2	Q-3	Q-1	Q-2	Q-3	Q-1	Q-2	Q-3
Homogeneous differential equation	Number	29	0	1	13	12	5	10	6	14	9	6	15
	Average	97%	0	3%	43%	40%	17%	33%	20%	47%	30%	20%	50%
Complete differential equation	Number	26	0	4	23	4	3	15	8	7	17	3	10
	Average	87%	0	13%	77%	13%	10%	50%	27%	23%	57%	10%	33%
First-order differential equation	Number	28	0	2	18	7	5	12	10	8	5	5	20
	Average	93%	0	7%	60%	23%	17%	40%	33%	27%	17%	17%	66%
Nonhomogeneous differential equation	Number	26	0	4	4	21	5	0	15	15	0	13	17
	Average	87%	0	13%	13%	70%	17%	0	50%	50%	0	43%	57%
Incomplete differential equation	Number	21	1	8	12	4	14	14	5	11	12	1	17
	Average	70%	3%	27%	40%	13%	47%	47%	17%	36%	40%	3%	57%
Bernoulli differential equation	Number	19	2	9	9	9	12	7	8	15	5	2	23
	Average	63%	7%	30%	30%	30%	40%	23%	27%	50%	17%	7%	76%

Research Findings

This study aimed to analyze the conceptual knowledge of some engineering students from the perspective of the connection and communication of mathematical concepts in differential equations. The scoring from the data analysis based on the framework mentioned in Table 1 is recorded in Table 2.

1. Q1: Number of students who gave the correct answer fully adhered to all four aspects.
2. Q2: Number of students who gave a partially correct answer, meaning they adhered to three of the four aspects at most.
3. Q3: Number of students who gave an incorrect answer, meaning they did not adhere to any of the four aspects correctly.

Analysis of Exam Questions:
Analysis of Question 1: Homogeneous Differential Equation

In solving the homogeneous differential equation, the student must first correctly identify the type of equation, apply the variable substitution, transform the equation into a separable differential equation, and finally solve the integral.

As shown in Table 2, 17% of students successfully applied the variable substitution $= \frac{y}{x}$. However, 47% of students were unsuccessful in transforming the equation into a separable differential equation. This stage demonstrates the student's weakness or failure in connecting and linking the concept of separable equations. Furthermore, 50% of students failed to solve the integral related to their previous mathematics course. This indicates that students struggle to connect the concepts from their first-year calculus course when dealing with integrals. In other words, while students were successful in the first stage of identifying the type of equation, they failed in the second and third stages (accuracy of mathematical relations and correct use of mathematical relations), particularly in substituting the variable u and solving the integral. Figure 1 shows an example of an unsuccessful student response.

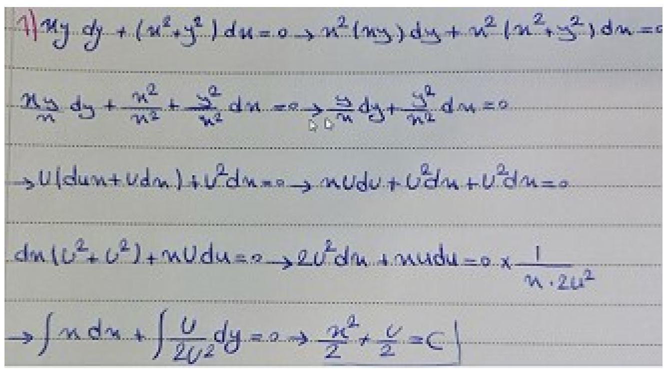

Figure 1: An example of an unsuccessful answer in solving a homogeneous differential equation

In other words, it can be said that the student failed in the following steps:

$$\frac{y}{x}dy + \left(1 + \frac{y^2}{x^2}\right)dx = 0$$

$$\left(u = \frac{y}{x}\right) \quad \Longrightarrow \quad u(xdu + udx) + (1 + u^2)dx = 0$$

$$uxdu + u^2dx + (1 + u^2)dx = 0$$

$$(u^2 + 1 + u^2)dx + uxdu = 0$$

$$(2u^2 + 1)dx + uxdu = 0$$

$$\frac{udu}{2u^2 + 1} + \frac{dx}{x} = 0$$

$$\int \frac{u}{2u^2 + 1}du + \int \frac{1}{x}dx = 0$$

$$\frac{1}{4}ln\ |2u^2 + 1\ | + ln\ |x\ | = c$$

Analysis of Question 2: Exact Differential Equation

In solving a complete differential equation, the completeness of the equation is first examined. Assuming that the following equation is a differential equation:

$$p(x,y)dx + q(x,y)dy = 0$$

If we have:

$$\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}$$

In this case, the differential equation is complete.

Students performed relatively well in solving the exact differential equation. According to Table 2, students correctly identified the type of equation and then partially correctly determined the partial derivatives of coefficients dx and dy, followed by finding $u(x,y)$. Therefore, it can be claimed that students could establish connections and communication regarding partial derivatives and polynomial integration. In other words, students successfully solved exact differential equations (identifying the type of equation, accuracy of mathematical relations, coherence in solving differential equation problems, and correct use of mathematical relations).

Analysis of Question 3: First-Order Differential Equation

The general form of a first-order differential equation is as follows:

$$y' + p(x)y = q(x)$$

Which is calculated using the following formula:

$$y = e^{-\int p(x)dx} \left(\int e^{\int p(x)dx} q(x)dx \ + c \right)$$

In solving question 3, which involves solving a first-order differential equation, students first needed to identify the type of equation, then find $P(x)$ and $Q(x)$. Finally, these are substituted into the formula to solve the integral. Figure 2 provides an example of an unsuccessful student response.

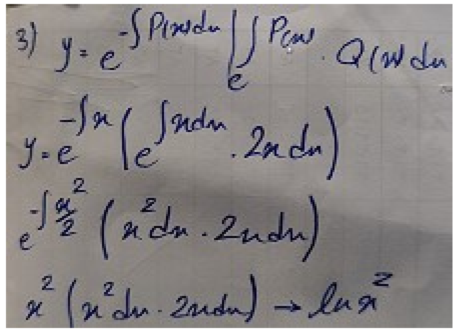

Figure 2: An example of an unsuccessful answer in solving the first-order differential equation

As shown in Table 2, in solving the first-order differential equation, students were somewhat successful in identifying P(x) and Q(x) Moreover, substituting them into the formula. However, 66% of students failed to solve the integral, indicating a significant weakness in connecting and linking the concepts from their first-year calculus course related to integrals and their integration methods. In other words, students successfully identified the equation and the accuracy of mathematical relations, meaning they correctly identified it as a first-order differential equation. However, they failed to use mathematical relations, specifically in integration methods, correctly.
In other words, it can be said that the student failed in the following steps:

$$y = e^{-\int xdx}(\int e^{\int xdx} 2xdx + c)$$
$$= e^{\frac{-x^2}{2}}(\int e^{\frac{x^2}{2}} 2xdx + c)$$

At this stage, the student should use the method of variable change.

$$y = e^{\frac{-x^2}{2}}\left(2\int e^u\, du + c\right)$$
$$= e^{\frac{-x^2}{2}}(2e^u + \mathrm{c})$$
$$= e^{\frac{-x^2}{2}}\left(2e^{\frac{x^2}{2}} + c\right)$$
$$= 2 + ce^{\frac{-x^2}{2}}$$

Analysis of Question Four: Non-Homogeneous Differential Equation

The general form of a non-homogeneous differential equation is as follows:

$$(a_1x + b_1y + c_1)dx + (a_2x + b_2y + c_2)dy = 0$$

First, it is converted to the following homogeneous equation:

$$(a_1X + b_1Y)dX + (a_2X + b_2Y)dY = 0$$

Then, it is solved like a homogeneous equation. In solving a non-homogeneous differential equation, students must first identify it as non-homogeneous, then convert it to a homogeneous equation, transform it into a separable differential equation, and finally solve the integral. Figure 3 shows an example of a student's response.

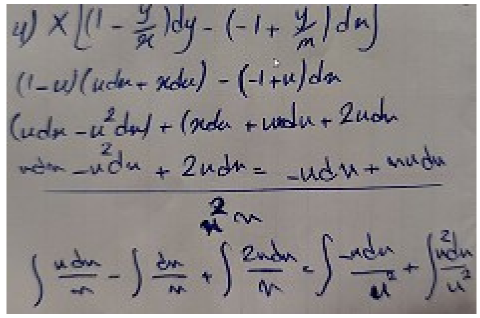

Figure 3: Example of an unsuccessful response to solving a non-homogeneous differential equation

According to Table 2, students successfully converted the non-homogeneous differential equation to a homogeneous one when solving the non-homogeneous differential equation. However, they were unsuccessful in converting to a separable differential equation (50%) and solving the integral (70%). Therefore, the student's weaknesses relate to solving separable differential equations from their first-year calculus course. The findings of this question about students' inability to convert non-homogeneous equations to homogeneous ones support the results from Question 1. It can be analyzed that students struggled to connect Question 1 (homogeneous equation) with Question 4 (non-homogeneous equation). In other words, while students were successful in the initial identification stage, they lacked coherence in the problem-solving process in the stages of verifying mathematical relations and correctly applying mathematical methods, indicating insufficient precision in solving algebraic relations.

In other words, it can be said that the student failed in the following steps:

$$
\begin{aligned}
&(1-u)(u\,dx + x\,du) - (1+u)dx = 0\\
&(1-u)u\,dx + (1-u)x\,du - (1+u)dx = 0\\
&(u - u^2 - 1 - u)dx + (1-u)xdu = 0\\
&-(1+u^2)dx + x(1-u)du = 0
\end{aligned}
$$

Ultimately, the method should be a separable differential equation where:

$$\frac{dx}{x} - \frac{(1-u)du}{(u^2+1)} = 0$$

Then, both sides should be integrated.

$$\int \frac{dx}{x} - \int \frac{(1-u)du}{(u^2+1)} = c$$

$$ln\ |x\ | + arctg\ u + \frac{1}{2} ln\ |u^2+1\ | = c$$

However, students performed incorrectly at all stages.

Analysis of Question Five: Incomplete Differential Equation

Method for solving an incomplete equation:

If the following equation is a differential equation:

$$p(x,y)dx + q(x,y)dy = 0$$

This equation is incomplete if it satisfies the following condition:

$$\frac{\partial p}{\partial y} \neq \frac{\partial q}{\partial x}$$

In this case, it must be completed, which can be done using one of the following formulas:

$$F(x) = e^{\int h(x)dx} \quad \text{where} \quad h(x) = \frac{\frac{\partial p}{\partial y} - \frac{\partial q}{\partial x}}{q(x,y)}$$

$$F(y) = e^{\int h(y)dy} \quad \text{where} \quad h(y) = \frac{\frac{\partial q}{\partial x} - \frac{\partial p}{\partial y}}{P(x,y)}$$

$$F(z) = e^{\int h(z)dz} \quad \text{where} \quad h(z) = \frac{\frac{\partial p}{\partial y} - \frac{\partial q}{\partial x}}{yq(x,y) - xP(x,y)}$$

In solving an incomplete differential equation, students must first recognize it as incomplete, then convert it to a complete equation, find the appropriate integrating factor, substitute it into the formula, and solve the integral. Figure 4 shows an example of a student's response.

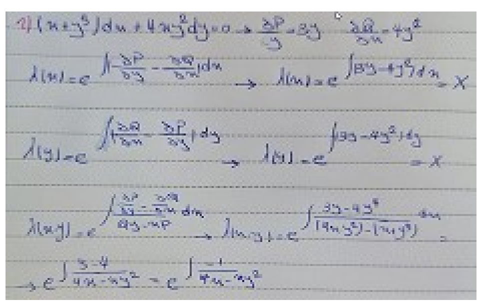

Figure 4: Example of an unsuccessful response to solving an incomplete differential equation

According to Table 4, 47% of students were unsuccessful in finding the appropriate integrating factor when solving the incomplete differential equation.

36% of students were unsuccessful in substituting into the integrating factor formula and solving the expression. Simplifying the fraction and factoring the resulting expression requires connecting to high school algebra. 57% of students were unsuccessful in solving the integral, indicating a lack of connection with their first-year calculus course and integration methods. In this question, since most students did not correctly find the integrating factor, they failed to solve the resulting complete differential equation. On the other hand, students were somewhat successful in solving Question 2, which involved a complete differential equation $u(x, y)$. This suggests that if faced with a complex problem in the final integration stage, students would be unable to obtain the correct answer. Therefore, it can be concluded that while students were successful in the initial identification stage, they were weak or unsuccessful in subsequent stages (verification of mathematical relations, coherence in solving differential equation problems, and correct application of mathematical methods).
In other words, it can be said that the student failed in the following steps:

$$\frac{\partial p}{\partial y} = 3y^2 \quad , \quad \frac{\partial q}{\partial x} = 4y^2$$

$$e^{\int \frac{3y^2-4y^2}{4xy^2}dx} = e^{\int -\frac{dx}{4x}}$$

$$= e^{-\frac{1}{4}ln\,|x\,|}$$

$$= \frac{1}{\sqrt[4]{x}} + c$$

Analysis of Question Six: Bernoulli Differential Equation

Method for solving Bernoulli's differential equation:
If the following equation is a Bernoulli equation:

$$y' + p(x)y = q(x)y^n$$

Which has been transformed from the following equation initially:

$$u' + p(x)u = q(x)$$

It is transformed into a first-order equation and then solved like a first-order one.
In solving the Bernoulli differential equation, students must first recognize it as a Bernoulli equation, then transform it into a first-order equation, substitute it into the formula, and solve the integral. Figure 5 shows an example of a student's response.

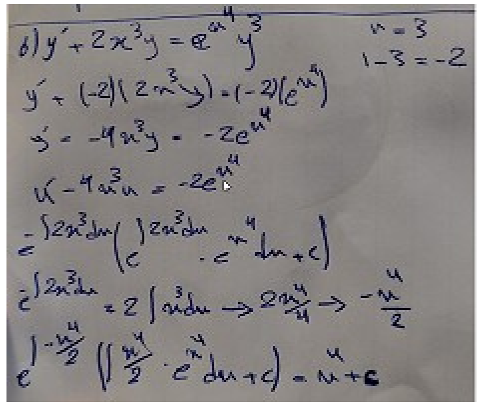

Figure 5: Example of an unsuccessful response to solving the Bernoulli differential equation

According to Table 2, in solving the Bernoulli differential equation, 40% of students were unsuccessful in converting the Bernoulli equation to a first-order equation. This indicates a lack of connection and understanding of first-order differential equations. Additionally, 50% of students were unsuccessful in substituting into the first-order equation formula, reflecting their weaknesses in handling algebraic expressions from high school. Furthermore, 76% of students were unsuccessful in solving the integral, showing their inability to connect with their first-year calculus course material.

From the analysis of Question 6, it can be inferred that due to the inadequate ability to transform the Bernoulli equation to a first-order equation, the results obtained in the later stages of this question corroborate the conclusions drawn in Question 3. In other words, while students were successful in the initial recognition stage, they were weak or unsuccessful in stages 2, 3, and 4 (verification of mathematical relations, coherence in solving differential equations, and correct application of mathematical methods).

For instance, in the expression:

$$u = y^{-2} \quad \Rightarrow \quad u' - 4x^3 u = -2e^{x^3}$$

Students were unsuccessful in substituting into the first-order equation formula.
In other words, it can be said that the student failed in the following steps:

$$u = e^{-\int -4x^3 dx}\left(\int e^{\int -4x^3 dx}\left(-2e^{x^4}\right)dx + c\right)$$

$$u = e^{x^4}\left(-2\int e^{-x^4} e^{x^4} dx + c\right)$$

$$= e^{x^4}\left(-2\int dx + c\right)$$

$$= e^{x^4}(-2x + c)$$

Ultimately, they were unsuccessful in solving the integral and obtaining the correct answer.

Conclusion

Academic success is a key component of social status, significantly influencing an individual's position regarding job structure and income (Khodaie, 2010). Success in mathematics courses is a prerequisite for academic achievement in specialized engineering subjects, to the extent that Kite (2010) considers student's academic success in engineering courses dependent on their abilities in mathematics. Concerns about academic decline sometimes adversely affect students (Lopes, 2012). The course on differential equations is crucial for many engineering students, both as a prerequisite for specialized courses and as an application in the second step of the modeling process (Karimi Fardinpour, 2012). However, various factors contribute to student's academic decline in this course.

One significant factor contributing to student's discouragement and academic decline in differential equations is their inability to connect and relate mathematical concepts to solving differential equations. The researchers of this study aimed to identify student's weaknesses, which stem from a lack of connection with previously learned concepts, by examining their performance in solving several differential equations. To this end, student's responses were evaluated in a few aspects: identifying the method for solving differential equations, accuracy in writing mathematical relationships and formulas, and maintaining coherence in solving differential equation problems. The results showed that the most significant weaknesses were in the stages of mathematical accuracy and the application of mathematical relationships (such as simplifying algebraic expressions, factoring, using integration methods, and solving integrals). Most students struggled with algebraic expressions related to high school mathematics and the integration methods taught in their prerequisite university course, Calculus 1. This suggests that student's weaknesses relate to university-level topics or indicate a foundational deficiency in their high school mathematics education.

The weaknesses in university-level mathematics are linked to previously taught topics in differential equations, such as separable differential equations. Additionally, students struggle to connect and relate to high school topics such as simplifying algebraic expressions, factoring, and using algebraic identities. These foundational weaknesses in high school algebra lead to difficulties connecting these concepts to more advanced topics.

Furthermore, the study found that students had difficulties using mathematical relationships correctly. They struggled with integration methods and, ultimately, with solving integrals, which hindered their ability to solve differential equations effectively. This weakness is traced back to their Calculus 1 course. The student's inability to solve even simple integrals, such as polynomial integrals that do not require special integration techniques, highlights their lack of proficiency. Consequently, they faced even greater challenges with more complex integration methods. Students who cannot correctly handle algebraic expressions will likely struggle to connect the necessary concepts for integration and select the appropriate integration method. It can be argued that students understand the concepts of differential equations well, as solving differential equations often follows algorithmic methods, which they can handle. However, when it comes to other mathematical relationships, such as algebraic manipulation, analysis, identities, and integration, they fail to connect these concepts properly, leading to errors.

The results of this study highlight the critical role of connection and communication in mathematics education, a focus emphasized by key educational frameworks such as the National Curriculum Framework in Iran and the National Council of Teachers of Mathematics (NCTM). Assessing students by emphasizing these factors is essential for fostering conceptual understanding. Supporting evidence from research demonstrates similar issues: Widjajanti (2013) identified struggles in connecting expressions with mathematical models, Putri and Wutsq (2017) found difficulties in applying mathematical concepts to real-world problems and Pambudi et al. (2020) highlighted the importance of linking mathematical skills to problem-solving. Junarti et al. (2019) also reported that students fail to relate mathematical elements within structures like group theory. Conducting studies to identify weaknesses in students' connections and coherence in other mathematics courses can complement this research. Additionally, school mathematics teachers should emphasize prerequisite concepts for university courses to reduce student's challenges in higher education. Professors can help students by emphasizing mathematical connections and coherence, enabling them to see mathematics as an interconnected web of concepts and skills rather than isolated topics, thus fostering meaningful learning. Considering students' weaknesses in the integration part of Mathematics 1, it would be beneficial for instructors to review integration methods at the beginning of the semester. This would enable students to connect better and apply these methods, ultimately improving their ability to establish a link between integration and solving equations.

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Appendix 1:

Differential equations test questions

Question	Differential equation type
1) $xy\,dy + (x^2 + y^2)\,dx = 0$	Homogeneous differential equation
2) $(2xy + 5)\,dx + (x^2 + 6y^3)\,dy = 0$	Complete differential equation
3) $y' + xy = 2x$	First-order differential equation
4) $y' = \dfrac{2 + x + y}{x - y}$	Nonhomogeneous differential equation
5) $(x + y^3)\,dx + 4xy^2\,dy = 0$	Incomplete differential equation
6) $y' + 2x^3y = e^{x^4}y^3$	Bernoulli differential equation